

NAME:

Math 150 Practice Exam 3.1

Instructions: WRITE YOUR NAME CLEARLY. Do as many problems as you can for a maximal score of 100. SHOW YOUR WORK!

1. Find a simple expression for $\int \left(4\sqrt{x} - \frac{4}{\sqrt{x}}\right) dx$ [10 pts]

2. Find a simple expression for $\int \sec 4\theta \tan 4\theta d\theta$ [10 pts]

3. Use geometry to evaluate $\int_0^4 \sqrt{16 - x^2} dx$ [10 pts]

4. Use Riemann sums to evaluate $\int_1^4 (x^2 - 1) dx$ [10 pts]

5. Compute $\lim_{n \rightarrow \infty} \frac{2}{n} \left(\sqrt{1 + 1 \frac{2}{n}} + \sqrt{1 + 2 \frac{2}{n}} + \cdots + \sqrt{1 + n \frac{2}{n}} \right)$

[10 pts]

6. Find $\frac{d}{dx} \int_{x^2}^{10} \frac{dz}{z^2+1}$

[10 pts]

7. Find the average value of the function $f(x) = x^3$ over the interval $[-1, 1]$
[10 pts]

8. Calculate $\int_0^{\pi/2} \sin^2 \theta \cos \theta \, d\theta$ [10 pts]

9. Find a simple expression for $\int t^3 \sin t^4 dt$ [10 pts]

10. Suppose that $\lim_{h \rightarrow 0} \frac{1}{h} \int_2^{2+h} f(t) dt = 5$. Compute $\lim_{h \rightarrow 0} \frac{1}{h} \int_2^{2-3h} f(t) dt$ [10 pts]

Extra-Credit

11. Under what conditions on $f(x)$ can the limit $\lim_{h \rightarrow 0} \frac{1}{h} \int_x^{x+h} f(t) dt$ be easily computed? Explain your answer. [10 pts]

12. Let $G(x) = \int_0^x \cos(s^2) ds$. Find $G'(x)$. [10 pts]

13. Show why $\sum_{k=1}^n k = \frac{n(n+1)}{2}$. [10 pts]

14. Suppose that f is an even function with $\int_0^8 f(x)dx = 9$. Evaluate $\int_{-1}^1 xf(x^2)dx$. [10 pts]