NAME:

Math 150 Exam 1

Instructions: WRITE YOUR NAME CLEARLY. Do as many problems as you can for a maximal score of 100. SHOW YOUR WORK!

1. True or False?

    a) For any function f, $\lim_{x \to a} f(x) = f(a)$ [2 pts]

    b) If $f(x) = 3^x$, then $f'(x) = x3^{x-1}$ [2 pts]

    c) $\lim_{x \to 0} \dfrac{Sin(3x)}{x} = 3$ [2 pts]

    d) $\dfrac{x^2 + x - 6}{x - 2} = x + 3$ for all $x \in (-\infty, \ \infty)$ [2 pts]

    e) $\lim_{x \to 4} \dfrac{5 + \sqrt{x}}{\sqrt{5 + x}} = \dfrac{7}{3}$ [2 pts]

2. Let $f(x) = -x^2 + 3x - 2$. Write the equation of the tangent line to the graph of f(x) at x = 1. (Hint: Use derivative "shortcuts")   [10 pts]

3.   Evaluate $\lim\limits_{x \to -\infty} \left(x + \sqrt{x^2 + 2x} \right)$ [10 pts]

4.   Let $f(x) = \sqrt{1 - 3x}$. Use the definition of the derivative to find $f'(x)$

[10 pts]

5. Use the Quotient Rule to differentiate $K(x) = \dfrac{Cos(x)}{1 - Sin(x)}$ [10 pts]

6. Suppose that $f(3) = -4$, $f'(3) = 1$, $g(3) = 5$, and $g'(3) = 2$. Compute $(fg)'(3)$.   [10 pts]

7.   Evaluate $\lim\limits_{x \to 0} \dfrac{Sin(2x)Sin(5x)}{x^2}$ [10 pts]

8.   Let $a > 0$ be a positive real number. Define $f(x) = \begin{cases} x^2 & if \ x < a \\ 3x & if \ x \geq a \end{cases}$.

What is the value of a if f is continuous on the entire real number line?   [10 pts]

9. A particle moves along the x-axis such that its position at time t is given by
$x(t) = -2te^t$.

     a) What is the particle's velocity at time t = 2?   [8 pts]

     b) Is the particle moving right or left?   [2 pts]

10. Let $f(x) = \dfrac{(x+1)(x^2+4)(x-7)^3}{(x+1)^2(x-7)^2}$.

     a) Determine the values of x for which f is continuous. Write your answer in interval notation.   [5 pts]

     b) For each x where the function is discontinuous, determine if the discontinuity is removable or not.   [5 pts]

Extra-Credit

11. Prove by means of a delta-epsilon argument that $\lim_{x \to -2}(x^2 - x) = 6$ [10 pts]